THE DETERMINATION OF RICHARDSON NUMBER AND ROUGHNESS PARAMETER AT OCEAN VESSEL "VICTOR"

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Submitted in partial fulfillment of
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IN
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ABSTRACT

Research on wind and temperature profiles over land have established a number of fundamental relationships that are tested here over an oceanic location. Most attempts at calculating the roughness parameter over oceanic surfaces have been based on surface-layer theories, and often employ an iterative procedure in approaching the sea surface. This paper employs a similarity relationship between the wind and temperature profiles, using data at ship "Victor" appropriate to forced convective theory, as defined by Priestley.

The theory of forced convection, as applied here, makes it necessary to compute the Richardson number and the Monin-Obukhov scale length as preliminaries to the computation of roughness parameter. The roughness parameter and friction velocity are obtained from the theory using an iterative procedure. The wind speed at 20 meters is correlated against scale length and roughness parameter. The latter makes very little contribution to the variance of the windspeed while the scale length contributes in a manner reasonably expected from theory.

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List of Symbols

Symbol	Definition
u_n	Wind speed at n meters height
Z	Height in centimeters above the water surface
k	Von Karman Constant
u^*	Friction velocity
L	Monin-Obukhov scale length
T^*	Scaling temperature
H	Eddy heat transport in the vertical
R_i	Richardson number
g	Acceleration of gravity (980 cms)
C_p	Specific heat of air
P	Density of air
Z_o	Roughness parameter
θ	Potential temperature
S	Non-dimensional wind shear after Monin & Obukhov
K_H	Eddy diffusivity for heat conduction
K_M	Eddy diffusivity for momentum flux in two directions
K_{6-20}	Ratio of wind at 600 cms to that at 2,000 cms.

Symbol	Definition
$K_{1.4}$	Ratio of $(Z \frac{\partial u}{\partial z})_{1.4}$ to the wind difference at 20 meters minus that at ten centimeters
τ	Surface stress
$C_{1.4}$	A more exact form of $K_{1.4}$
V_g/fZ_o	Surface Rossby number

1. Introduction

Studies of the roughness parameter, Z_0, have been numerous over land and a number of fundamental relationships using various meteorological parameters have been determined. Far less research has been accomplished in this field over the oceans. This is readily understandable when one considers the expense and difficulty in erecting and maintaining an instrumented platform at sea. With this in mind, one could turn to what should be a prime source of data--the ocean station weather ships.

Certain problems become evident very quickly when considering station Victor. Four different ships occupy this station and only two have the same anemometer height. This, at first, seemed to present an insurmountable problem in using the data but the theoretical error involved in using a median height of twenty meters instead of actual heights is less than two and one half percent. When considering that winds are recorded only to the nearest knot and temperature to the nearest degree, the error introduced in the wind profiles by considering all heights equal to twenty meters is insignificant.

Monin and Obukhov [3], in their universal similarity theory introduced three scaling parameters that are considered essentially invariant with height for layers near the ground. These three

parameters are scaling velocity, u^*, scaling length, L, and scaling temperature, T^*, and have been well described in a recent text (Lumley and Panofsky) [1] on atmospheric turbulence. L, u^* and T^* are defined as follows:

$$L = -\frac{u^{*3} C_p \rho T}{k g H} \tag{1}$$

$$T^* = -\frac{H}{k u^* C_p \rho} \tag{2}$$

$$u^* = \sqrt{\frac{\tau}{\rho}} \tag{3}$$

It should be noted that a selection of cases has been limited to cases for which $-Ri \leq .032$ holds. According to Priestley's criterion [4] our restriction limits the study to the cases of "forced convection". Here Ri refers to the turbulent parameter

$$Ri = \frac{g}{\theta} \frac{\partial\theta/\partial Z}{(\partial U/\partial Z)^2} \tag{4}$$

and is to be distinguished from the flux Richardson number Rf, defined as

$$Rf = \frac{K_H}{K_M} Ri \tag{5}$$

Under normal circumstances, it is considerably easier to deal with Ri than with Rf, since the factor K_H / K_M is itself an increasing function of Ri.

2. Important properties of the forced-convective surface layer

According to the similarity theory for the near-neutral, or forced convection case (as defined in section 1), the wind profile can be written

$$U = \frac{u^*}{k}\left(\ln\frac{Z}{Z_0} + \frac{\beta' Z}{L}\right) \tag{6}$$

In a like manner the temperature profile can be written, after Lumley and Panofsky

$$\theta - \theta_0 = T^*\left(\ln\frac{Z}{Z_0} + \frac{\beta' Z}{L}\right) \tag{7}$$

In both (6) and (7), Lumley and Panofsky [1] suggest the value $\beta' = 4.5$.

By dividing equation (7) by equation (6), and taking the derivative of both numerator and denominator with respect to height, one has a relationship governing the constants T^* and u^*/k :

$$\frac{T^*}{\left(\frac{u^*}{k}\right)} = \frac{\partial\theta/\partial Z}{\partial U/\partial Z} \tag{8}$$

Relationship (8) means that the change in the temperature over a given height is proportional to the change in the wind over the same height. Even more important, this ratio is constant with height since T^*, u^*, and k are all considered constant in the surface layer.

The ratio of these constants $T^* / (u^*/k)$ may be determined from the air temperature at psychrometer height (six meters) and the temperature at Z_o, assumed to be the same as the sea surface temperature, so that equation (8) may be written, in finite-difference form centered at three meters as

$$\frac{T^*}{\left(\frac{u^*}{k}\right)} = \frac{\theta_6 - \theta_0}{u_6 - u_0} \tag{9}$$

Here u_o is the wind speed at Z_o and is zero. By equation (8) this ratio is constant at all levels in the surface layer, including that at 1.41 meters.

From the definition of L, it follows that L satisfies the identity

$$\frac{Z}{L} = \frac{K_H}{K_M} Ri\ S \tag{10}$$

However, Priestley [4] has introduced a surface layer scale lenght L' given by

$$L' = L \frac{K_H}{K_M} \tag{11}$$

so that (10) becomes

$$\frac{Z}{L'} = Ri\,S \tag{12}$$

a formula lacking the troublesome ratio K_H/K_M, which has been found to depend upon Ri. But from the definition of T^*, we have

$$\frac{-H}{C_p \rho T^*} = \frac{u^{*2}}{(u^*/k)}$$

or

$$\frac{K_H (\partial\theta/\partial Z)}{T^*} = \frac{K_M\, \partial u/\partial Z}{u^*/k} \tag{13}$$

From (13), together with the similarity profiles (6) and (7), which apply in the case of forced convection, it follows that $K_H=K_M$, and therefore L=L'. This is a fundamental property characteristic of similarity profiles in forced convection.

3. Derivation of a specific value for $(Ri)_{1.41}$

From equations (4) and (9), the Richardson number is expressible in the form

$$Ri = \frac{gZ}{u_6}\frac{(\theta_6-\theta_0)}{\theta_0}\,\frac{1}{(Z\,\partial u/\partial Z)_{1.41}} \tag{14}$$

where every parameter in (14) is now known except for the quantities u_6 and $(Z\frac{\partial u}{\partial Z})_{1.4}$.

The surface-layer wind profile is assumed to be valid to heights of twenty meters. The windspeed u_6 may then be related to u_{20} by a new parameter through

$$u_6 = u_{20} K_{6-20} \tag{15}$$

where K_{6-20} is formed from the ratio of u_6 to u_{20}. This now leaves the quantity $(Z \frac{\partial u}{\partial z})_{1.41}$ as the sole unknown in equation (14). One may approximate this quantity by forming the ratio of two wind differences centered at 1.41 meters. In the following equations, subscripts denote the height of a variable in meters although substitution in centimeters is generally required. For example we have

$$\left(Z \frac{\partial u}{\partial z}\right)_{1.41} \doteq \frac{\Delta u}{\Delta \ln Z} = \frac{u_2 - u_1}{\ln 2} \tag{16}$$

and

$$\frac{\frac{u_2 - u_1}{\ln 2}}{\frac{u_{20} - u_{0.1}}{\ln 200}} = \frac{\left(1 + \frac{450}{L \ln 2}\right)}{\left(1 + \frac{8955}{L \ln 200}\right)} \tag{17}$$

Therefore, it follows that

$$\frac{u_2 - u_1}{\ln 2} \doteq u_{20} K_{1.4} \tag{18}$$

Now the last two unknowns in equation (14) are found in terms of known or measured quantities. Following Martin [2],

$$K_{1.4} = \frac{\left(1 + \frac{450}{L \ln 2}\right)}{\left(1 + \frac{8955}{L \ln 200}\right)} \tag{19}$$

and, similarly,

$$K_{6-20} = 1 - \frac{\left(\ln \frac{20}{6} + \frac{6300}{L}\right)}{\left(\ln \frac{200}{Z_0} + \frac{8955}{L}\right)} \tag{20}$$

with all values in centimeters. K_{6-20} may be approximated satisfactorily for obtaining Ri by omitting $\ln Z_0$.

The right sides of (15) and (16) are now known as a function of L so that (14) may be solved for Ri as

$$Ri = \frac{gZ}{K_{1.4}(L)\,K_{6-20}(L)}\,\frac{(\theta_6 - \theta_0)}{\theta_0\,U_{20}^2} \tag{21}$$

where $K_{1.4}$, and K_{6-20} are given as functions of L in (19), (20).

In the forced convection regime, Lumley and Panofsky show that the Richardson number may be written as

$$Ri = \frac{\frac{Z}{L}}{1 + \frac{18Z}{4L}} \quad , \quad K_H = K_M \tag{22}$$

Eliminating Ri between (21) and (22) leads to a quadratic in L, the only meaningful root of which is given by

$$L = \frac{A + (A^2 - C)^{1/2}}{B} \tag{23}$$

$$\begin{aligned} A &= 1.49(\ln 200) - .7 \ln 10/3 - 9000\, Ri'(\ln 200 + \ln 2000) \\ B &= 2g\, Ri'(\ln 200)(\ln 2000) \\ C &= 4[(g Ri')^2(\ln 200)(\ln 2000)(81 \times 10^6) \\ &\qquad - 900\, g\, Ri'(\ln 200)(\ln 2000)] \end{aligned} \tag{24}$$

where

$$Ri' = \frac{g\,(\theta_6 - \theta_0)}{\theta_0\,U_{20}^2}$$

Ri may now be obtained by solving equation (22) with Z=141.1 cm and the value of L obtained from (23).

The quadratic equation defined by (23) and (24) had as input data only windspeeds at 20 meters, and temperature - differences measured from the surface to 6 m. Only one data-set was selected at ship Victor per day, and these were selected with a view toward insuring cases of forced convection: for example, a vertical temperature difference less than or equal to 6°C, and windspeeds as large as possible under these circumstances were selected. A table of Ri values computed as a function of both u_{20} and $\theta_6 - \theta_0$ is included in the Appendix.

4. Iterative procedure of solution of roughness parameter

Following Martin [2], use is made of equation (6) to obtain the quantity $u_{20} - u_{0.1}$ in the form

$$U_{20} - U_{0.1} = \frac{U^*}{k}(2.3026)(2.30103) + \frac{8955}{9000}\left(\frac{9000}{L}\right)\left(\frac{U^*}{k}\right) \tag{25}$$

But with u_{20} as follows

$$U_{20} = 2.3026\left(\frac{U^*}{k}\right)\log_{10}\frac{2000}{Z_0} + \left(\frac{U^*}{k}\right)\left(\frac{9000}{L}\right) \tag{26}$$

one obtains the result, $U_{0.1}$, given in (27)

$$U_{0.1} = .005\,U_{20} + \frac{U^*}{k}(2.2988)\left(1 - \frac{2.2911}{2.2988}\log_{10} Z_0\right) \tag{27}$$

By using the Monin-Obukhov normalized wind shear

$$S = \frac{k}{U^*} Z \frac{\partial U}{\partial Z}$$

and (12), it follows that

$$\frac{U^*}{k} = \left(\frac{\partial U}{\partial \ln Z}\right)_{1.4}\left(\frac{L\,Ri}{Z}\right)_{1.4} = \left(\frac{U_2 - U_1}{\ln 2}\right)\left(\frac{L\,Ri}{Z}\right)_{1.4} \tag{28}$$

Because Z_0 is small compared to 20 meters, the quantity $(\partial U/\partial \ln Z)_{1.4}$ will be rewritten involving a larger span, as follows

$$\frac{\frac{U_2 - U_1}{\ln 2}}{U_{20} - U_{0.1}} = C_{1.4}$$

where $C_{1.4}$ will be defined immediately below.

$$\left(\frac{\Delta U}{\Delta \ln Z}\right)_{1.4} \doteq \frac{U_2 - U_1}{\ln 2} = C_{1.4}\,(U_{20} - U_{0.1})$$

From (6), we then have

$$C_{1.4} = \frac{U_2 - U_1}{\ln 2} \Big/ (U_{20} - U_{0.1}) = \frac{1}{\ln 200} \frac{\left(1 + \frac{649.2}{L}\right)}{\left(1 + \frac{1681.4}{L}\right)} \tag{29}$$

Combining (28) and (29) leads to

$$\frac{U^*}{k} = \left(\frac{L\,Ri}{Z}\right)_{1.4} C_{1.4}\,(U_{20} - U_{0.1}) \tag{30}$$

where $C_{1.4}$ is defined in (29).

A third equation used in the iteration-process is

$$U_{0.1} = \frac{U^*}{k}(2.3026)(1 - \log_{10} Z_0)\left(1 + \frac{45}{L}\right) \tag{31}$$

but this has already been built into equation (25). The latter may be used for more accurate determination of Z_o once an earlier iterate, or satisfactory guess, has been established. Equation (31) assumes the use of equation (6) to an elevation of 10 cm above mean sea level.

To adopt an efficient iterative method which does not lay too much emphasis on the last 10 cm, we must give proper weight to the layer 0-20 m. Here, we use equations (27) and (30) with $u_{o.1}$ eliminated. This leads to the result

$$\frac{u^*}{k} = \frac{C_{1.4}\left(\frac{LRi}{Z}\right)_{1.4}(0.995)}{1 - C_{1.4}\left(\frac{LRi}{Z}\right)_{1.4}\left(2.2988 - 2.2911\log_{10} Z_0\right)} \tag{32}$$

and, of course, we also have

$$U_{20} = \frac{u^*}{k}\left(\ln 200 + \ln\frac{10}{Z_0} + \frac{9000}{L}\right) \tag{33}$$

The iteration process may be summarized as follows:

1. Assume $Z_o^{(1)} = 0.1$ and obtain a corresponding estimate of $(u^*/k)^{(1)}$ from (32).

2. With this first estimate of $(u^*/k)^{(1)}$, use equation (33), with u_{20} and L known, to determine $Z_o^{(2)}$.

3. Repeat the procedure using $Z_o^{(2)}$ in (32) to get a second estimate of $(u^*/k)^{(2)}$.

4. Repeat the iteration until a final residue is obtained:

$$\left| Z_o^{(n)} - Z_o^{(n-1)} \right| \leq 10^{-4}$$

When this has been accomplished Z_o is taken to be $Z_o^{(n)}$, and $(u^*/k)^{(n)}$ is the appropriate value from either (32) or (33).

One safeguard must be followed in the use of (32): the denominator of the right side must not be allowed to become zero or negative. However, the use of (33) helps to restrict (u^*/k) to reasonable, non-negative values.

The results, giving Z_o, u^*/k as well as u_{20}, Ri', L and Ri are listed in Table 1 (Appendix). In addition, some statistical relationships between the parameters are also considered in the following section. The raw data considered was drawn from 30 randomly selected dates. The value of L is computed from equation (23), while $Ri_{1.41}$ is obtained from (22).

The computations were made using data cards obtained from the National Weather Records Center, Asheville, North Carolina, for ocean station "Victor". All cards selected were for 0300Z, the observation nearest noon local time, from the months of November and December of 1960 and of March 1961. The number of intervals between successive days was considered a random function of the date. Wind values were tabulated in this paper in centimeters per second, and temperatures in degrees Celsius. Sea conditions during the period were not considered, with the exception that no major storms were in the immediate area.

5. Results and conclusions

The values of u_{20}, L and Z_o for the sample of 30 cases were subjected to a multiple regression analysis leading to a result of form

$$U_{20} = -30.118 Z_o - 0.00531 L + 757.97 \tag{34}$$

variable 3 variable 1 variable 2

Of the two independent variables tested in (34), Z_o plays an insignificant part in that its partial regression coefficient was -0.00969. When the mean value of $\bar{Z}_o = 0.1066$ cm was inserted into (34), the simple regression equation

$$U_{20} = -0.00531L + 754.76 \tag{35}$$

resulted. The variable L had a partial correlation coefficient of -0.56153, so that its effect upon the reduction of the variance of u_{20} was sufficiently strong that resulting F-value due to the regression involving L was 6.576. This F-value, with 2 and 27 degrees of freedom, is significant at a level in excess of the 99 percent confidence level. This result is not surprising from the manner in which 1/L enters u_{20}, Eq. 33. A brief summary of some of the other statistics obtained are listed:

$\bar{U}_{20} = 987.07$ cms	$\bar{Z}_o = 0.10662$ cm	$\bar{L} = -43{,}773.6$ cm
$\sigma_u = 378.47$ cms	$\sigma_{Z_o} = 0.10238$ cm	$\sigma_L = 40{,}682.6$ cm
	$r_{31\cdot2} = -0.00961$	$r_{32\cdot1} = -0.56153$

The most surprising fact regarding the statistics, and equation (34) is the complete unimportance of the variable Z_o in describing u_{20}. At first glance, equation (33) would suggest a much stronger dependence, since $-\ln Z_o$ is one of the terms of the right side, and this should indicate that u_{20} decreases with increasing Z_o. This is a result presented by many writers, who emphasize the tendency for u to decrease with increasing surface Rossby number Vg/fZ_o. However, from equation (33) it can be seen that

$$U_{20} = \left(\frac{U^*}{k}\right)_n \frac{\left(1+\frac{649.2}{L}\right)\left(1+\frac{\ln\frac{10}{Z_o}}{\ln 200}+\frac{1681.4}{L}\right)}{\left(1+\frac{1681.4}{L}\right)} \tag{36}$$

where $(u^*/k)_n$ is the neutral value of (u^*/k). That part of (u^*/k) which depends upon stability is primarily incorporated in the ratio

$$\left(1+\frac{649.2}{L}\right)\Big/\left(1+\frac{1681.4}{L}\right)$$

so that the neutral part of (u^*/k) is essentially

$$\left(\frac{U^*}{k}\right)\Big/C_{1.4}$$

The final point to be emphasized here then is as follows: Within equation (36) the term $-\ln Z_o$ tends to reduce u_{20}. However, carefully measured results of Hay (1955) [5] indicate that

$$(U^{*2})_n = 13.1 g Z_o$$

so that u^* may be expected to increase in the boundary layer as Z_o increases. Obviously these two effects oppose one another, with a wind-increasing tendency from (u^*/k) and a wind-decreasing tendency arising from $-\ln Z_o$ within the bracket of (36). This oddity resulting from opposite effects produced by an increase in Z_o appears to explain the lack of correlation of u_{20} with Z_o.

No statistical analysis involving u^*/k with Z_o has been made but their values are listed in the Appendix, and may serve as the basis of future research on the subject.

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APPENDIX I

Table of selected micrometeorological parameters in the surface layer.

ư_	ΔT(°C)	Ri'(10^{-8})	L	Ri	u*/k	Z_o
1081.5	-3.88	-1.15	-25197.329	-.0057	112.510	.098703
412.0	-4.44	-9.02	-4900.552	-.0327	56.065	.250295
309.0	-1.66	-6.00	-6399.332	-.0243	135.457	.34632
1030.0	-3.88	-1.26	-23040.770	-.0063	107.509	.060164
1339.0	-2.22	-0.43	-64212.942	-.0022	171.079	.709676
1081.5	-2.77	-0.82	-34474.195	-.0041	111.440	.01238
1442.0	-2.77	-0.46	-59822.933	-.0024	146.968	.145697
515.0	-1.11	-1.44	-20410.493	-.0071	54.025	.126881
1442.0	-6.11	-1.01	-28242.996	-.0050	149.440	.06137
1133.0	-2.77	-0.75	-37639.751	-.0038	116.493	.042618
618.0	-2.22	-1.98	-15405.752	-.0095	65.760	.095000
1133.0	-1.11	-0.29	-92272.089	-.0015	114.868	.111753
2060.0	-2.22	-0.18	-150915.860	-.0009	208.067	.158484
875.5	-4.44	-1.98	-15426.860	-.0095	93.154	.089827
772.5	- .55	-0.32	-85614.909	-.0017	78.378	.098703
618.0	-1.11	-0.99	-28764.428	-.0049	64.009	.035126
721.0	-3.88	-2.55	-12423.038	-.0119	77.801	.208904
875.5	-1.66	-0.74	-37875.237	-.0038	55.825	.174716
1133.0	-2.79	-0.74	-37980.775	-.0038	116.468	.098703
1030.0	-2.22	-0.71	-39240.355	-.0036	105.800	.098703
721.0	-3.33	-2.18	-14163.219	-.0103	98.529	.738132
1133.0	-0.56	-0.15	-184229.295	-.0008	104.304	.062660
1133.0	-3.89	-1.02	-29043.613	-.0051	117.4444	.098703
1287.5	-2.78	-0.56	-49075.979	-.0029	147.914	.315771
1081.5	-1.11	-0.32	-85030.331	-.0017	109.737	.018613
1236.0	-1.11	-0.25	-110239.246	-.0013	125.113	.057789
1545.0	-6.11	-0.86	-32759.215	-.0044	148.858	.087884
772.5	-1.11	-0.63	-44052.566	-.0032	79.153	.098703
618.0	-1.67	-1.48	-19944.649	-.0073	37.485	.023398
721.0	-3.89	-2.54	-12462.568	-.0118	65.775	.103646